Maximizing Revenue from Strategic Recommendations under Decaying Trust
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Abstract
Suppose your sole interest in recommending a product to me is to maximize the amount paid to you by the seller for each successful recommendation. How should you recommend optimally if I become more inclined to ignore you with each irrelevant recommendation you make?

We prove that even if the recommendee regains her initial trust on each successful recommendation, the expected revenue the recommender can make over an infinite period due to payments by the seller is bounded. This can only be overcome when the recommendee also incrementally regains trust during periods without any recommendation.

Classification of Advertising Schemes
• Addressing: Personal vs. general. A recommendation is per se more personal than an advertisement and should be adapted to reflect the individual needs and interests of the potential buyer.
• Trust: High vs. low. A recommendation should come from someone the potential buyer trusts and feels loyal or close to. This can be a personal friend or maybe a well-respected blogger.
• Intention: Altruistic vs. commercial. The intention of a recommendation by a friend is generally not commercial and she recommends something as she believes you would profit from it.

Examples
1. Billboards for a pizza restaurant.
   Addressing: general, trust: low, intention: commercial.
2. Sponsored search results.
   Addressing: personal, trust: low, intention: commercial.
3. Book review on Amazon.
   Addressing: general, trust: high, intention: altruistic.
4. Recommending a friend a laptop.
   Addressing: personal, trust: high, intention: altruistic.

The Setting: Revenue Maximizing Recommendations
For each of \( n \) products the recommender has two options, “recommend” or “not recommend.” A recommendation is successful if the buyer buys the product, in which case the recommender gets a reward \( r \). Initially, the probability \( p \) of success is \( p_0 < 1 \). With each unsuccessful recommendation \( p \) drops to \( p = l \cdot p \), where the factor \( l < 1 \) corresponds to a loss in trust.

Extensions: \( p \) can increase again in two ways.
1. \( p \) is reset to \( p_0 \) on each successful recommendation.
2. Each time the recommender does not recommend anything, \( p = \min(g \cdot p, p_0) \) for a factor \( g \geq 1 \).

The recommender’s sole goal is to maximize the expected reward summed over all \( n \) products.

Without Reset, without Recovery
When \( g = 1 \) (i.e., no recovery) and the probability of success is not reset to \( p_0 \) on a successful recommendation, the optimal strategy is to recommend all products. Solving the corresponding infinite sum analytically gives the following, bounded expression.

\[
R(p_0, l) = \frac{p_0}{1 - p_0} \cdot \frac{1}{1 - l} \cdot r < \infty
\]

With Reset, without Recovery
We prove that even when a single successful recommendation resets the buyer’s trust to \( p_0 \), the total expected revenue is bounded.

**Theorem 1.** Let \( \log(\alpha) = \int_1^\alpha \frac{1}{1-t} \, dt \), \( c = \max(p_0, l) \), and \( \delta(c) = (1 - c) \exp(\log(1 - c) / \ln(c)) \).

\[
R(p_0, l) \leq \frac{1 - \delta(c)}{\delta(c)} \cdot r < \infty
\]

With Reset, with Recovery
Here we study the case where every \( k \)-th product is recommended.

If \( k \) is too large the revenue is sub-optimal though, ultimately unbounded:

**Theorem 2.** Let \( \psi \) be the smallest integer such that \( l \cdot g^\psi \geq 1 \). If \( k > \psi \), then, for all \( 1 > p_0, l > 0 \) and \( \infty > g \geq 1 \),

\[
A^{(k)}(p_0, l, g) = \left[ \frac{n}{k} \right] \cdot p_0 \cdot r.
\]

If \( k \) is too small the revenue is bounded:

**Theorem 3.** Let \( \psi \) be the smallest integer such that \( l \cdot g^\psi \geq 1 \). If \( k \leq \psi \), then, for all \( 1 > p_0, l > 0 \) and \( \infty > g \geq 1 \), there exist \( c_0 \) and \( \ell \) such that

\[
\lim_{n \to \infty} A^{(k)}(p_0, l, g) = \lim_{n \to \infty} M_n(p_0, \ell) < \infty.
\]

Simulation
Without any recovery
The total expected reward converges with or without recovery (green & red).

With recovery
For the “aggressive” heuristic (purple) which recommends every second item the total expected reward converges.

The other settings have an asymptotically unbounded reward.

Open Research Questions
• How exactly do web users respond to being shown irrelevant advertisements? Is it possible to revive their interest in banner ads?
• What are “optimal” auction mechanisms for sponsored search when the CTRs are non-constant and decay with each irrelevant advertisement being shown?