How much is your personal recommendation worth?

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Problem

You recommend a product to your friend. He buys it. How much should the seller pay you?

Extended Model

Input:
- Seller \( s = 0 \), recommenders \( R = \{1, \ldots, n\} \)
- Margin \( \delta \) (given by seller)
- Arguments \( A = \{a_1, \ldots, a_k\} \), arguments \( B_i \subseteq A \) used by \( i \in R \) (given by recommender)
- For each subset \( B \subseteq A \) a value \( v(B) \):
  \[ v(B) = \text{Pr}[\text{purchase when arguments in } B \text{ are used}] \cdot \delta \]

Output:
- Payoff vector \( x = (x_0, \ldots, x_n) \) such that \( x_i \geq 0 \), \( x_0 = 0 \) if \( i \in R \) and \( B_i = \emptyset \), and \( \sum_{i>0} x_i = v(\cup_i B_i) \)

Basic Model

Input:
- Seller \( s = 0 \), recommenders \( R = \{1, \ldots, n\} \)
- Margin \( \delta \) (given by seller)
- For each subset \( S \subseteq N = \{0, \ldots, n\} \) a value \( v(S) \):
  \[ v(S) = \text{Pr}[\text{purchase when recommended by } i \in S \setminus \{s\}] \cdot \delta \]
  [Assumption: \( v(S) = 0 \) if \( s \not\in S \)]

Output:
- Payoff vector \( x = (x_0, \ldots, x_n) \) such that \( x_i \geq 0 \) and \( \sum_i x_i = v(N) \)

Results

1. No pricing scheme with \( \sum_{i>0} x_i > 0 \) is truthful for the seller.
2. Pricing scheme using Shapley value is fair, but not truthful for the recommenders.
3. Pricing scheme using anonymity-proof Shapley value is fair, and truthful for the recommenders.

Shapley Value

The Shapley value \( \phi(v) = (\phi_0(v), \ldots, \phi_n(v)) \) is given by

\[
\phi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(|N|-|S|-1)!}{|N|!} \cdot (v(S \cup \{i\}) - v(S))
\]

Example

Seller \( s = 0 \), recommenders \( R = \{1, 2\} \)

Non-zero values:

- \( v(\{0\}) = 0.1 \cdot \delta \)
- \( v(\{0, 1\}) = (0.1 + 0.1) \cdot \delta \)
- \( v(\{0, 2\}) = (0.1 + 0.5) \cdot \delta \)
- \( v(\{0, 1, 2\}) = (0.1 + 0.1 + 0.5) \cdot \delta \)

Shapley value:

- \( \phi_{a_1}(v) = (0.1 + (0.1 + 0.5)/2) \cdot \delta = 0.4 \cdot \delta \)
- \( \phi_{a_2}(v) = 0.2/2 \cdot \delta = 0.05 \cdot \delta \)
- \( \phi_{a_3}(v) = 0.5/2 \cdot \delta = 0.25 \cdot \delta \)

Anonymity-Proof Shapley Value

For any set \( B = \cup_i B_i \) of declared arguments the anonymity-proof Shapley value \( \psi_a(v) \) for \( a \in B \) is:

\[
\psi_a(v) = \frac{\phi_a(v)}{\sum_{b \in B} \phi_b(v)} \cdot v(B)
\]

References: